Topic 9: Negative Exponents

for use after ***Accentuate the Negative*** **Investigation 4**

You have used positive exponents to represent repeated multiplication. Negative exponents are used to represent repeated division.

Problem 9.1

A. You can use patterns to find the value of negative exponents.

1. Copy and complete the table below.

Expression	Value
2^4	
2^3	
2^2	

2. What pattern do you see as you go from 2^4 to 2^3 and from 2^3 to 2^2?

3. The value of 2^8 is 256. Explain how to find the value of 2^7 using division.

B. **1.** Extend the table in part (A). Find $2^1, 2^0, 2^{-1}, 2^{-2}, 2^{-3}$, and 2^{-4}. Express values less than 1 as fractions.

2. You can use multiplicative inverses to find the value of an expression with a negative exponent. Use your answers in Part A (4) and Part B (1) to complete each statement.

$2^{-2} = \left(\frac{1}{2}\right)^{\blacksquare}$

$2^{-3} = \left(\frac{1}{2}\right)^{\blacksquare}$

$2^{-4} = \left(\frac{1}{2}\right)^{\blacksquare}$

3. Rewrite each expression using its reciprocal and a positive exponent. Then find the value of the expression.

a. 4^{-2}

b. $\left(\frac{2}{5}\right)^{-3}$

Exercises

For Exercises 1–6, find the value of each expression.

1. $(2)^{-1}$ **2.** $(3)^{-5}$ **3.** $(5)^{-3}$

4. $\left(\frac{2}{5}\right)^{-1}$ **5.** $\left(\frac{8}{3}\right)^{-3}$ **6.** $\left(\frac{7}{10}\right)^{-2}$

7. Anthony and Alexandra found the value of $(6)^3 \times (3)^{-2}$ in different ways. Are both methods correct? Why?

Anthony's Method:

I will rewrite using positive exponents. Then I will simplify the powers and multiply.

$$\begin{aligned}(6)^3 \times (3)^{-2} &= (6)^3 \times \left(\tfrac{1}{3}\right)^2 \\ &= 216 \times \tfrac{1}{9} \\ &= 24\end{aligned}$$

Alexandra's Method:

I will write positive exponents as repeated multiplication and negative exponents as repeated division. Then I will perform the operations.

$$\begin{aligned}(6)^3 \times (3)^{-2} &= [6 \times 6 \times 6] \div 3 \div 3 \\ &= 216 \div 3 \div 3 \\ &= 72 \div 3 \\ &= 24\end{aligned}$$

For Exercises 8–13, find the value of each expression.

8. $2 \times (4)^{-2}$ **9.** $\left(\frac{1}{5}\right)^{-3} \times \frac{4}{5}$ **10.** $\left(\frac{1}{2}\right)^{-4} \times \left(\frac{2}{7}\right)^{-2}$

11. $(5)^{-2} \times (10)^2$ **12.** $(5)^{-3} \times \left(\frac{3}{5}\right)^{-4}$ **13.** $\left(\frac{3}{8}\right)^{-3} \times \left(\frac{2}{3}\right)^{-4}$

For Exercises 14–19, find the value of each expression. Write values less than 1 using decimals.

14. $2 \times (10)^2$ **15.** $2 \times (10)^{-2}$ **16.** $7 \times (10)^{-3}$

17. $7 \times (10)^3$ **18.** $2.5 \times (10)^5$ **19.** $2.5 \times (10)^{-5}$

20. For parts (a) and (b), find the value of each expression.

a. 5^4 5^3 5^2 5^1 5^0

b. 10^4 10^3 10^2 10^1 10^0

c. For parts (a) and (b), look for a pattern. Use the pattern to predict the value of any positive number with exponent zero.

At a Glance

Topic 9: Negative Exponents

PACING 1 day

Mathematical Goals

- Simplify expressions containing negative exponents

Teaching Guide

Negative exponents are difficult for many students to understand. Some students may simply write a negative sign in front of their solution. It is necessary to explain to the students why negative exponents represent repeated division.

Positive exponents have already been covered in this course. A class discussion to review this topic would be beneficial at this time. By reviewing the fact that positive exponents indicate repeated multiplication, it will become apparent to students why negative exponents indicate repeated division. For example, since $2^3 = 2 \times 2 \times 2$ and $2^2 = 2 \times 2$, decreasing the exponent by 1 is equivalent to dividing by the base, 2. At this point, it may also be helpful to review the fact that any base raised to the zero power is equal to 1. Therefore, negative exponents involve repeated division.

After Problem 9.1A, have students choose another number and make a table of the values of that number raised to powers from 3 to -3.

Summarize Problem 9.1B by asking:

- *In what other mathematical situations do we use reciprocals?*
- *Why does it make sense to use reciprocals when finding the value of an expression with a negative exponent?*

Homework Check

When reviewing Exercise 7, ask:

- *Does each method work for solving the problem? What are some advantages and disadvantages of each method?*

Summarize Exercises 14–19 by asking:

- *How is knowing powers of 10 useful in measurement?*
- *How could some of these exercises be used to describe metric units?*

Assignment Guide for Topic 9

Core 1–19
Advanced 20

Answers to Topic 9

Problem 9.1

A. **1.** 16; 8; 4

2. Answers may vary. Sample: 2^3 is half of 2^4, and 2^2 is half of 2^3. Each term is one half of the previous term.

3. $2^8 = 2^7 \times 2$, so $2^7 = 2^8 \div 2$; divide 256 by 2.

B. **1.** $2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

2. 2; 3; 4

3. **a.** $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

b. $\left(\frac{5}{2}\right)^3 = \frac{125}{8}$

Exercises

1. $\frac{1}{2}$

2. $\frac{1}{243}$

3. $\frac{1}{125}$

4. $\frac{5}{2}$

5. $\frac{27}{512}$

6. $2\frac{2}{49}$

7. Yes; dividing by 3 twice is the same as multiplying by $\frac{1}{9}$.

8. $\frac{1}{8}$

9. 100

10. 196

11. 4

12. $\frac{5}{81}$

13. 96

14. 200

15. 0.02

16. 0.007

17. 7,000

18. 250,000

19. 0.000025

20. **a.** 625; 125; 25; 5; 1

b. 10,000; 1,000; 100; 10; 1

c. The value of any positive number with exponent zero is equal to 1.